On action-angle variables of 2-dimensional Toda field theories

V. S. Gerdjikov[1,2]

[1] *Institute of Mathematics and Informatics*
Bulgarian Academy of Sciences,
Acad. Georgi Bonchev Str., Block 8, 1113 Sofia, Bulgaria

[1] *Institute for Advanced Physical Studies*
111 Tsarigradsko chaussee, Sofia 1784, Bulgaria

We construct generalized Fourier transforms that which linearize the 2-dimensional Toda field theories related to the simple Lie algebra $\mathfrak{g}$ [1, 2, 3]. The corresponding Lax operator takes the form:

$$L\psi \equiv i\frac{\partial \psi}{\partial x} + \left(i\vec{q}_x - \lambda \sum_{\alpha\in\delta_0} E_\alpha \right) \psi(x,t,\lambda) = 0,$$

where $\vec{q}(x,t) = \sum_{j=1}^{r} q_j H_j \in \mathfrak{h}$ is an element of the Cartan subalgebra and δ_0 is the set of admissible roots of $\mathfrak{g}$. The operator L satisfies $\mathbb{Z}_h$ reductions of Mikhailov type defined by Coxeter automorphism of $\mathfrak{g}$. We find that the continuous spectrum of L fills up $2h$ rays starting from the origin of the complex λ-plane $l_\nu \equiv \arg\lambda = \pi/h$, $\nu = 0, 1, \dots, 2h-1$. The minimal set of scattering data is defined by the reflection coefficients on the rays l_0 and l_1. We derive the action-angle variables in terms of these reflection coefficients.

In fact these generalized Fourier transforms are spectral decompositions of the recursion operators related to L.

The dressing factors for the soliton solutions will be analyzed. Special attention will be paid to the exceptional Lie algebra $\mathfrak{g}_2$.

References

[1] V. S. Gerdjikov, A. B. Yanovski. *On soliton equations with $\mathbb{Z}_h$ and $\mathbb{D}_h$ reductions: conservation laws and generating operators.* J. Geom. Symmetry Phys. **31**, 57–92 (2013).

[2] V. S. Gerdjikov, A B Yanovski. Riemann-Hilbert Problems, families of commuting operators and soliton equations Journal of Physics: Conference Series **482** (2014) 012017.

[3] V S Gerdjikov, A B Yanovski. CBC systems with Mikhailov reductions by Coxeter Automorphism. I. Spectral Theory of the Recursion Operators. Studies in Applied Mathematics **134** (2), 145–180 (2015).