Quasifinite fields of prescribed characteristic and Diophantine dimension

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Abstract

Let $\mathbb{P}$ be the set of prime numbers, $\overline{\mathbb{P}}$ the union $\mathbb{P} \cup \{0\}$, and for any field E, let $\mathrm{char}(E)$ be its characteristic, $\mathrm{ddim}(E)$ the Diophantine dimension of E, $\mathcal{G}_E$ the absolute Galois group of E, and $\mathrm{cd}(\mathcal{G}_E)$ the Galois cohomological dimension $\mathcal{G}_E$. The research presented in this paper is motivated by the open problem of whether $\mathrm{cd}(\mathcal{G}_E) \leq \mathrm{ddim}(E)$. It proves the existence of quasifinite fields $\Phi_q \colon q \in \mathbb{P}$, with $\mathrm{ddim}(\Phi_q)$ infinity and $\mathrm{char}(\Phi_q) = q$, for each q. It shows that for any integer $m > 0$ and $q \in \overline{\mathbb{P}}$, there is a quasifinite field $\Phi_{m,q}$ such that $\mathrm{char}(\Phi_{m,q}) = q$ and $\mathrm{ddim}(\Phi_{m,q}) = m$. This is used for proving that for any $q \in \overline{\mathbb{P}}$ and each pair $k, \ell \in (\mathbb{N} \cup \{0, \infty\})$ satisfying $k \leq \ell$, there exists a field $E_{k,\ell;q}$ with $\mathrm{char}(E_{k,\ell;q}) = q$, $\mathrm{ddim}(E_{k,\ell;q}) = \ell$ and $\mathrm{cd}(\mathcal{G}_{E_{k,\ell;q}}) = k$. Finally, we show that the field $E_{k,\ell;q}$ can be chosen to be perfect unless $k = 0 \neq \ell$.

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