

Institute of Mathematics and Informatics  
Bulgarian Academy of Sciences



**International Workshop**  
**“Invariant Distances and Metrics**  
**in Complex Analysis”**

in honour of our teacher and friend Prof. Dr. Peter Pflug  
on the occasion of his 80th birthday

**July 11-15, 2023, Sofia, Bulgaria**

in the frame of the  
Fourth International Conference  
**MATHEMATICS DAYS IN SOFIA**

<https://mds.math.bas.bg/w-idmca/>



## Organizers



Institute of Mathematics and Informatics, Bulgarian Academy of Sciences  
Union of the Bulgarian Mathematicians  
Faculty of Mathematics and Informatics, Sofia University "St. Kliment Ohridski"

## Sponsors



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Education and Science

## Aim and Scope

The workshop brings together researchers around a central theme of the geometric properties of various distances and metric of domains in  $C_n$  (squeezing function, Gromov hyperbolicity, visibility of geodesics) as well as related subjects.

## Organising Committee

Nikolai Nikolov (IMI BAS, Bulgaria)

Pascal J. Thomas (Université Paul Sabatier, France)

Krassimira Ivanova (IMI BAS, Bulgaria)

## Speakers

Ahmed Yekta Ökten, Université Paul Sabatier, France (PhD student)

Anand Chavan, Jagiellonian University, Poland (PhD student)

Andrew Zimmer, University of Wisconsin-Madison, USA

Armen Edigarian, Jagiellonian University, Poland

Filippo Bracci, Università di Roma "Tor Vergata", Italy

Gautam Bharali, Indian Institute of Science, India

Hervé Gaussier, Université Grenoble 1, France

Łukasz Kosinski, Jagiellonian University, Poland

Matteo Fiacchi, University of Ljubljana, Slovenia

Sławomir Dinew, Jagiellonian University, Poland

Włodzimierz Zwonek, Jagiellonian University, Poland

Zywomir Dinew, Jagiellonian University, Poland

## Venue

The workshop will be held at the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences (Sofia, Acad. Georgi Bonchev Str., Block 8), Hall 503.

# Program Schedule

## 12th July (Wednesday)

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Chair: Gautam Bharali

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14:00	Filippo Bracci: <i>Visibility and geodesic loops in Kobayashi complete hyperbolic and Gromov hyperbolic domains</i>
14:45	Matteo Fiacchi: <i>The Julia-Wolff-Carathéodory theorem in convex domains of finite type</i>
15:30	Coffee break

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Chair: Filippo Bracci

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16:00	Gautam Bharali: <i>The squeezing function: exact computations and a new application</i>
16:45	Armen Edigarian: <i>Isometries on the symmetrized bidisc</i>
18:00	Welcome party of the workshop

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## 13th July (Thursday)

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Chair: Armen Edigarian

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14:00	Włodzimierz Zwonek: <i>2-proper holomorphic images of the Cartan domains</i>
14:45	Łukasz Kosinski: <i>Extension property</i>
15:30	Coffee break

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Chair: Włodzimierz Zwonek

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16:00	Sławomir Dinew: <i>Complex nonlinear equations, weak solutions and admissible domains</i>
16:45	Zywomir Dinew: <i>Aspects of B-regularity</i>
20:00	Official dinner, Hyatt Regency Sofia Hotel

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## 14th July (Friday)

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Chair: Pascal J. Thomas

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14:00	Herve Gaussier: <i>Invariant metrics – old and more recent results</i>
14:45	Ahmed Yekta Ökten: <i>Estimates of the Kobayashi distance of strongly pseudoconvex domains</i>
15:30	Coffee break

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Chair: Herve Gaussier

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16:00	Andrew Zimmer: <i>Bounded geometry in several complex variables</i>
16:45	Open problems
18:00	Sightseeing tour – Sofia historical center

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## Abstracts

### Ahmed Yekta Ökten

#### *Estimates of the Kobayashi distance of strongly pseudoconvex domains*

We will present sharp upper and lower bounds (of the same type) for the Kobayashi distance of a  $C^{2,\alpha}$ -smooth strongly pseudoconvex domain. These bounds come from the study of the behaviour of complex geodesics and extend the well-known Balogh-Bonk estimate (2000). The talk is based on a joint work with L. Kosinski and N. Nikolov.

### Andrew Zimmer

#### *Bounded geometry in several complex variables*

I will discuss several different bounded geometry conditions for a complex manifold. Then describe some applications.

### Armen Edigarian

#### *Isometries on the symmetrized bidisc*

We study Caratheodory isometries in the symmetrized bidisc. We show that any  $C^1$ -isometry is holomorphic or anti-holomorphic. By-pass we give also a new proof of the description of automorphisms of the symmetrized bidisc, using „only” formulas for the distance and infinitesimal metric, but not using W. Kaup’s result on the orbit of the origin under automorphisms (as it was done by Jarnicki-Pflug).

### Filippo Bracci

#### *Visibility and geodesic loops in Kobayashi complete hyperbolic and Gromov hyperbolic domains*

In this talk I will summarise the recent results (and open questions) about visibility and geodesic loops in bounded domains of the complex space which are both complete Kobayashi hyperbolic and Gromov hyperbolic. In particular, I will give a characterization of visibility and the existence of geodesic loops in terms of “geodesic frames” and show some applications.

## **Gautam Bharali**

### *The squeezing function: exact computations and a new application*

In recent years, work on the squeezing function has focused largely on a diagnostic for detecting (local) strong Levi pseudoconvexity. Comparatively less attention has been focused on what the squeezing function can say about the complex geometry of a domain and on the very amusing — but rather hard — problem of computing the squeezing function exactly. In this talk, we shall revisit the latter aspects of the squeezing function. This examination of the squeezing function will rely on an old idea, suitably adapted, of “fitting” large polydiscs into domains, which has been useful in the past in the non-strongly pseudoconvex context. The talk is based on a joint work with Diganta Borah and Sushil Gorai.

## **Herve Gaussier**

### *Invariant metrics – old and more recent results*

Invariant metrics under the action of biholomorphisms, such as the Kobayashi and Carathéodory metrics, or the Bergman and Kähler-Einstein metrics, are important metrics that can encode certain properties of the complex manifolds on which they are defined. In this talk, I will address some problems concerning their study. I will present some classical results establishing links between the geometric properties of complex manifolds and the properties of their invariant metrics. I will also present more recent studies in this direction.

## **Łukasz Kosinski**

### *Extension property*

We study sets in convex domains that have the polynomial extension property.

## **Matteo Fiacchi**

### *The Julia-Wolff-Carathéodory theorem in convex domains of finite type*

The classical Julia-Wolff-Carathéodory theorem says that, if  $f$  is a holomorphic self-map of the unit disk  $D$  and  $\xi \in \partial D$  such that  $\liminf_{z \rightarrow \xi} (1 - |f(z)|)/(1 - |z|) < \infty$  then the derivative  $f'$  has non-tangential limit at  $\xi$  and the limit value can be computed in terms of the Poincaré distance. The theorem has been generalized to several complex variables by Rudin in the unit ball, by Abate in strongly convex and by Abate-Tauraso in convex domains of finite type, under some technical assumptions.

In this talk I will present a version of the Julia-Wolff-Carathéodory theorem in convex domains of finite type in full generality, using metric geometry techniques. This is a joint work with L. Arosio.

## **Sławomir Dinew**

### *Complex nonlinear equations, weak solutions and admissible domains*

We report on recent progress in the study of nonlinear elliptic equations in domains in  $\mathbb{C}^n$ . Existence of weak solutions to those, if suitable boundary behavior is assumed, forces geometric properties of the domain.

## **Włodzimierz Zwonek**

### *2-proper holomorphic images of the Cartan domains*

The ideas that generalize the construction of the symmetrized bidisc and the tetrablock as 2-proper holomorphic images of the Cartan domains will be presented. The new domains have many geometric properties that will be presented in the lecture. That will contain description of automorphisms, properties of the Bergman kernel, (non)-convexity of the domains. The talk is based on the common research with Gargi Ghosh.

## **Zywomir Dinew**

### *Aspects of B-regularity*

The notion of B-regularity was introduced by Sibony in his famous Duke paper from 1987. For compact sets it means that one can approximate uniformly an arbitrary continuous function by continuous plurisubharmonic functions defined on (varying) neighborhoods. We will present some new properties of B-regular compact sets and domains and provide some examples. This is an ongoing research with Sławomir Dinew.

# MDS 2023 Plenary Talks

## Hyatt Regency Sofia Hotel

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### 10th July

10:00-10:50

Vesselin Dimitrov: *Converse theorems and Diophantine analysis*

11:10-12:00

Kevin Ford: *Detecting prime numbers in general sequences*

### 11th July

10:00-10:50

Bojko Bakalov: *Algebras in 2-dimensional conformal field theory*

11:10-12:00

Plamen Iliev: *Fejér-Riesz factorizations and Bernstein-Szegő measures*

### 12th July

10:00-10:50

Gueo Grantcharov: *Isotropic Killing vector fields and complex surfaces*

11:10-12:00

Sławomir Dinew: *Geodesics in the space of convex and plurisubharmonic functions*

### 13th July

10:00-10:50

Pavel Boychev: *Letting the steam off STEAM education (Developing and utilizing intuition, imperfection and gamification in STEAM)*

11:10-12:00

Stoyan Mihov: *Finite-state machines and neural networks for language modelling*

### 14th July

10:00-10:50

Bojan Popov: *Invariant domain preserving approximations for hyperbolic systems and related problems*

11:10-12:00

Peter Minev: *Efficient integrators for incompressible flow*