FAMILIES OF SUBSPACES OF TOPOLOGYCAL SPACES OF COUNTABLE TYPE

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The present work has been started jointly with Mitrofan Choban and continues his ideas on families of subspaces possessing collective properties. The general concept of a metrizable family of subspaces of a topological space was introduced, studied and applied to the Selection theory for multivalued mappings by M. Choban (see [2]). It generalizes ideas of P. Kenderov in the case of Topological groups and such of V. Geiler for Uniform spaces. Further research was done on families of p-subspaces, complete p-subspaces and $\mathcal{A}(\mathcal{P})$-subspaces by M. Choban, E. Mihaylova (see [3]) and on first countable families by E. Mihaylova (see [4]).

Families of subspaces of Q-countable type of a topological space are defined and studied in the present work. They are a generalization of the notion of a countable type space. Topological spaces of countable type were defined and examined by A. Arhangel'skii (see [1]).

Definition 1 Let $\mathcal{Q} \in \{k, s\}$ where k is the property compactness and s is the property countable compactness. A family $\mathcal{A}$ of subspaces of a space X is called a family of $\mathcal{Q}$-countable type if for every $\mathcal{A}$-balanced compact subset H of X with the property $\mathcal{Q}$ there exists a compact $\mathcal{A}$-balanced subset F of X such that $H \subseteq F$ and $\chi_X(F, \mathcal{A}) = \aleph_0$ (the character of F in X relatively to the family $\mathcal{A}$).

Theorem 1 Let $i \in \{2, 3, 3.5\}$, $\mathcal{A}$ be a family of subspaces of a T_i-space X, $\mathcal{Q} \in \{k, s\}$ and $X = \cup\mathcal{A}$. The following assertions are equivalent:

1. $\mathcal{A}$ is a family of subspaces of $\mathcal{Q}$-countable type.
2. There exist a T_i-space Z, a continuous pseudometric d on Z and an open continuous mapping $f : Z \longrightarrow X$ of the space Z onto the space X such that $\mathcal{L} = \{f^{-1}(L) : L \in \mathcal{A}\}$ is $\mathcal{Q}$-metrizable by the pseudometric d and f is $(\mathcal{L}, \mathcal{Q})$-covering.

Application in the Theory of selections of multivalued mappings is obtained:

Theorem 2 Let $\theta : Y \longrightarrow X$ be a lower semicontinuous mapping of a paracompact F_σ-discrete space Y into a space X.

Fix a non-empty family $\mathcal{A}$ of $\mathcal{Q}$-countable type of non-empty subspaces of the topological space X.

Denote $\mathcal{A}^* = \{H \subseteq L : L \in \mathcal{A}, H \neq \varnothing\}$, the family of all $\mathcal{A}$-subsets of the space X.

1. If $\theta(y) \in \mathcal{A}^*$ for every $y \in Y$, then there exists an upper semicontinuous mapping $\psi : Y \longrightarrow X$ such that:
   - $\psi(y) \cap \theta(y) \neq \varnothing$ and $\psi(y)$ is a subspace with property $\mathcal{Q}$ for every $y \in Y$;
   - if $L \in \mathcal{A}$, $y \in Y$ and $\psi(y) \cap L \neq \varnothing$, then $\psi(y) \subset L$.
2. If $\theta(y) \in \mathcal{A}$ for every $y \in Y$, then there exists an upper semicontinuous mapping $\psi : Y \longrightarrow X$ such that $\psi(y) \subset \theta(y)$ and $\psi(y)$ is a subspace with property $\mathcal{Q}$ for every $y \in Y$.

References

[1] A. Arhangel'skii, *A class of spaces which contains all metric and all locally compact spaces*, Matem. Sb. 67 (1965), 55-88.

[2] M. Choban, *Reduction theorems on the existence of continuous sections. Sections over subsets of factor spaces of topological groups*, Matem. Issledovania 8, no. 4, 1973, Chişinev: Ştiinţa, 111-156.

[3] M. Choban and E. Mihaylova, *Special families of subspaces and its applications*, Accepted.

[4] E. Mihaylova, *Open images of metrizable families*, Mathematica Balkanica, 21, 3-4, (2007), 407-420.