

A DIOPHANTINE PROBLEM WITH A PRIME AND THREE SQUARES OF PRIMES OF A SPECIAL FORM

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ABSTRACT. We prove that if $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are non-zero real numbers, not all of the same sign, λ_1/λ_2 is irrational, and η is any real number then, for any $\delta > 0$, the inequality $|\lambda_1 p_1 + \lambda_2 p_2^2 + \lambda_3 p_3^2 + \lambda_4 p_4^2 + \eta| < (\max p_j)^{-1/18+\delta}$ has infinitely many solutions in prime variables p_1, p_2, p_3, p_4 such that, for each $i \in \{1, 2, 3, 4\}$, $p_i + 2$ has at most 126 prime factors. The proof uses Davenport - Heilbronn adaption of the circle method together with a vector sieve method.

Keywords: Rosser's weights, vector sieve, circle method, almost primes, diophantine inequality.

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