MODULAR FORMS ON BALL QUOTIENTS, BIRATIONAL TO BIELLIPTIC SURFACES

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Let $\Gamma < \mathrm{SU}_{1,2}$ be a neat lattice, whose associated quotient $\mathbb{B}/\Gamma$ of the complex 2-ball $\mathbb{B} = \mathrm{SU}_{1,2}/\mathrm{S}(\mathrm{U}_1 \times \mathrm{U}_2)$ is birational to a bi-elliptic surface Y. The reported work obtains the dimensions of the spaces $[\Gamma, n]_o$ of the cuspidal Γ-modular forms of weight $n \geq 2$ and the dimensions of the spaces $[\Gamma, n]$ of all Γ-modular forms of weight $n \geq 2$. It gives a sufficient condition for a subspace $V \subset [\Gamma, n]$ to provide a regular projective morphism $\Phi_V : \widehat{\mathbb{B}/\Gamma} \to \mathbb{P}(V)$ of the Baily-Borel compactification $\widehat{\mathbb{B}/\Gamma}$ of $\mathbb{B}/\Gamma$, as well as a sufficient condition for Φ_V to be a regular projective embedding.

The general results are applied on an example $\mathbb{B}/\Gamma_{\sqrt{-1}}$ of Di Cerbo and Stover from 2019, which is birational to $\left(E_{\sqrt{-1}} \times E_{\sqrt{-1}}\right)/\mathbb{Z}_2$ with $E_{\sqrt{-1}} = \mathbb{C}/\left(\mathbb{Z} + \mathbb{Z}\sqrt{-1}\right)$ and associated with a lattice $\Gamma_{\sqrt{-1}}$, commensurable with the Picard modular group $\mathrm{SU}_{1,2}(\mathbb{Z}[\sqrt{-1}])$ over the Gaussian integers $\mathbb{Z}[\sqrt{-1}]$. Namely, the $\Gamma_{\sqrt{-1}}$-modular forms of weight 1 are shown to be of dimension 3 and to determine a non-regular, finite, dominant rational map $\Phi_{[\Gamma_{\sqrt{-1}},1]} : \widehat{\mathbb{B}/\Gamma_{\sqrt{-1}}} \dashrightarrow \mathbb{P}^2(\mathbb{C})$. The reported work constructs a subspace $V_2 \subset [\Gamma_{\sqrt{-1}}, 2]$ of $\dim_{\mathbb{C}} V_2 = 7$, which gives rise to a regular embedding $\Phi_{V_2} : \widehat{\mathbb{B}/\Gamma_{\sqrt{-1}}} \to \mathbb{P}^6(\mathbb{C})$ and subspaces $V_{2n} \subset [\Gamma_{\sqrt{-1}}, 2n]$ of $\dim_{\mathbb{C}} V_{2n} = 8$ for $\forall n \geq 2$, defining regular embeddings $\Phi_{V_{2n}} : \widehat{\mathbb{B}/\Gamma_{\sqrt{-1}}} \to \mathbb{P}^7(\mathbb{C})$.

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