Tensor Decompositions And Applications To Optimization Problems And Data Analysis

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Abstract

We review the main tensor decompositions for discrete tensors, as: Canonical Polyadic, Tucker, Tensor Train, Tensor Chain, Hierarchical, and extend them in a general decomposition scheme, known as tensor network. For a given connected graph with n nodes and d open edges, we can decompose a given d-order tensor $\mathcal{T} \in \Re^{n_1 \times \cdots \times n_d}$, as simultanious contructions of n component tensors of smaller dimensions, which contruct along the common indices indicated by the common edges. The above mentioned decompositions correspond to very simple adjacency matrices of the corresponding graphs, which leads to huge dimensionality reduction of the initial tensor (from exponential to linear in d). We demonstrate the usefulness of such decompositions for solving huge optimization problems, which are decomposed to similar ones on the cores of the decomposition (and therefore have much smaller dimension), and solved alternatively. Such decompositions have analogues for the continuous case, which correspond to representing a function of several variables by means of functions of less variables. Main tools for such representations are the Singular Value Decomposition (for the discrete case) and the Schmidt decomposition (for the continuous case). As examples we present solving huge linear systems and eigenvalue problems (known), linear programming and conjugate gradient method (new). Another application is a variant of the Kernel Principal Component Analysis, which produces a dimensionality reduction twice: first by a tensor decomposition and second by the kernel embedding.

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