FREE BICOMMUTATIVE SUPERALGEBRAS

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We introduce the class of bicommutative superalgebras over an arbitrary field of characteristic different from 2. The class consists of all nonassociative $\mathbb{Z}_2$-graded algebras A with a vector space decomposition $A = A_0 \oplus A_1$, satisfying the polynomial superidentities of left- and right-supercommutativity

$$x(yz) = (-1)^{\overline{x}\,\overline{y}} y(xz) \text{ and } (xy)z = (-1)^{\overline{y}\,\overline{z}} (xz)y,$$

where $\overline{u} \in \{0, 1\}$ is the parity of the homogeneous element $u \in A_0 \cup A_1$.

We present an explicit construction of the free bicommutative superalgebras, find their bases as vector spaces and show that they share many properties typical for ordinary bicommutative algebras [1, 2, 3] and commutative associative superalgebras. In particular, in the case of free algebras of finite rank we compute the Hilbert series, establish an analogue of the classical Hilbert Basissatz for two-sided ideals and see that the Gröbner-Shirshov bases of these ideals are finite. Concerning problems studied in the theory of varieties of algebraic systems, we prove that the variety of bicommutative superalgebras satisfies the Specht property. In the case of characteristic 0 we compute the sequences of cocharacters and codimensions.

References

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