

CURVES ON CONES GIVING RISE TO NON-REDUCED COMPONENTS OF THE HILBERT SCHEME OF CURVES

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ABSTRACT

In our work [CIK21] we showed the existence of generically smooth components of the Hilbert scheme of smooth projective curves $\mathcal{I}_{2g-4\gamma+2,g,r}$ for $\gamma \leq r \leq g - 3\gamma + 2$. The general point of such a component turned out to be curve on a cone over a curve of genus γ and degree $g - 2\gamma + 1$ in \mathbb{P}^{r-1} , which was cut by a general quadratic hypersurface. In the present report we consider curves on a cone that pass through its vertex and are also triple covers of the base of the cone, which is a general smooth curve of genus γ and degree e in $\mathbb{P}^{e-\gamma}$. Using the free resolution of the ideal of such a curve found by Catalisano and Gimigliano in [CG99], and a technique involving very flat families introduced by Ciliberto in [Cil87], we show that the deformations of such curves remain on cones over a deformation of the base curve. This allows us to prove that for every $\gamma \geq 3$ and $e \geq 4\gamma + 5$ there exists a non-reduced component \mathcal{H} of the Hilbert scheme $\mathcal{I}_{3e+1,3e+3\gamma,e-\gamma+1}$ of smooth curves of genus $3e + 3\gamma$ and degree $3e + 1$ in $\mathbb{P}^{e-\gamma+1}$. We show that $\dim T_{[X]}\mathcal{H} = \dim \mathcal{H} + 1 = (e - \gamma + 1)^2 + 7e + 5$ for a general point $[X] \in \mathcal{H}$. The reported results are based on the paper [CIK23]. The last developed further our approach aimed at constructing generically smooth and non-reduced components of the Hilbert scheme of curves in projective spaces of high dimension introduced in [CIK21] and [CIK22].

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