Parametric Variational Principles in Banach Spaces

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We consider optimization problems depending on a parameter and investigate under which conditions the solutions of the perturbed problems with small perturbations depend in a "good" way on the parameter. The word "good" here has several different meanings: the solution is 1) a continuous function of the parameter, 2) a measurable function of the parameter, 3) a Carathéodory function of the parameter, 4) a function which is continuous on a dense G_δ subset of its domain. We provide different sufficient conditions for "good" dependence in all of these cases. The perturbations are similar to those in the Ekeland variational principle and in the Borwein-Preiss variational principle.

We consider also a parameterized variational inequality (A, Y) in a Banach space E defined on a closed, convex and bounded subset Y of E by a monotone operator A depending on a parameter. We prove that under suitable conditions, there exists an arbitrarily small monotone perturbation of A such that the perturbed variational inequality has a solution which is a continuous function of the parameter, and is near to a given approximate solution. In the nonparametric case this can be considered as a variational principle for variational inequalities, an analogue of the Borwein-Preiss variational principle. Some applications are given: 1) an analogue of the Nash equilibrium problem, defined by a partially monotone operator, 2) a variant of the parametric Borwein-Preiss variational principle for Gateaux differentiable convex functions under relaxed assumptions and 3) a generisity result in sense of porosity stating that the most variational inequalities considered here are well posed (which means that the complement of the set of the well posed variational inequalities is σ-porous).

The tool for proving the main result is a useful lemma about existence of continuous ε-solutions of a variational inequality depending on a parameter. It has an independent interest and allows a direct proof of an analogue of Ky Fan's type inequality for monotone operators, introduced here, which leads to a new proof of the Schauder fixed point theorem.

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