EXTREMALS TO NEW GAGLIARDO-NIRENBERG INEQUALITY AND SOLITARY WAVES

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Abstract. We study the extremals of a new Gagliardo-Nirenberg inequality with radial symmetry and involving slow decaying potentials. We analyse the existence of standing wave solutions for the Cauchy problem

$$\begin{cases} i\partial_t u + \Delta u + W(|x|)u \pm V(t,x,|u|^2)u|u|^{\alpha-2} = 0, \quad (t,x) \in \mathbb{R} \times \mathbb{R}^d, \\ u(0,x) = u_0(x). \end{cases}$$

Here we shall assume $0 \leqslant W(|x|) \lesssim |x|^{-\rho}$, with $\rho \in (0,2)$, $V : \mathbb{R} \times \mathbb{R}^d \times \mathbb{R}_+ \to \mathbb{R}_+$ and such that $V(t,x,|u|^2)u|u|^{\alpha-2}$, with $\alpha > 2$, is an H^1-subcritical nonlinearity, either of local or non-local type. Some eventual applications and open problems will be also discussed.

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