SMOOTH TOROIDAL COMPACTIFICATIONS OF BALL QUOTIENTS WITH BI-ELLIPTIC MINIMAL MODEL

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Let $\mathbb{B} = \left\{ [z_0 : z_1 : z_2] \in \mathbb{P}^2(\mathbb{C}) \,|\, |z_0|^2 - |z_1|^2 - |z_2|^2 > 0 \right\}$ be the complex 2-ball and $\Gamma < \mathrm{SU}_{1,2}$ be a lattice with non-compact quotient $\mathbb{B}/\Gamma$. Suppose that the toroidal compactification $X = (\mathbb{B}/\Gamma)'$ of $\mathbb{B}/\Gamma$ is a smooth surface and Y is a minimal model of X. In 2017 Di Cerbo and Stover show that there are exactly five smooth toroidal compactifications $X = (\mathbb{B}/\Gamma)'$ with minimal Euler number $e(X) = 1$. One of them has an abelian minimal model Y, while the other four are birational to bi-elliptic surfaces.

The reported work classifies the smooth toroidal compactifications $X = (\mathbb{B}/\Gamma)'$ with Euler number $e(X) = 2$ or 3. It turns out that all of them have an abelian or a bi-elliptic minimal model Y. We recognize the co-abelian such X, which cover an example of Hirzebruch from 1984 or an example of Holzapfel from 2001. Among the smooth $X = (\mathbb{B}/\Gamma)'$ with $e(X) \in \{2, 3\}$ and a bi-elliptic minimal model, we distinguish the ones, which are induced by series of examples of Di Cerbo and Stover, published in articles from 2018 and 2019.

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