The $(\overline{N}, p, q)$ summability and its applications to Tauberian theory

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Abstract

Let $p = (p_m), q = (q_n)$ be two sequences of nonnegative numbers such that $p_0,\ q_0 > 0$ and

$$P_m := \sum_{i=0}^{m} p_i \to \infty \quad \text{and} \quad Q_n := \sum_{j=0}^{n} q_j \to \infty \ \text{ as } \ m, n \to \infty.$$

The weighted means of (s_{mn}) determined by the weight sequences (p_m) and (q_n) are defined by

$$\sigma_{mn} := \frac{1}{P_m Q_n} \sum_{i=0}^{m} \sum_{j=0}^{n} p_i q_j s_{ij},$$

where $P_m Q_n > 0$ for all $(m, n) \in \mathbb{N} \times \mathbb{N}$ (see [4]). A double sequence (s_{mn}) is called $(\overline{N}, p, q)$ summable to a finite number L if $P\text{-}\lim_{m,n\to\infty} \sigma_{mn} = L$.

In this study, we are interested in relation between the $(\overline{N}, p, q)$ method and P-convergence for double sequences. In this context, we generate some Tauberian conditions, controlling one-sided and two-sided oscillatory behavior of a double sequence with respect to (P_m) and (Q_n), from the $(\overline{N}, p, q)$ summability to P-convergence with some restrictions on the weight sequences p and q. We demonstrate that

$$\frac{P_m}{p_m} \Delta_{10} u_{mn} = O(1) \quad \text{and} \quad \frac{Q_n}{q_n} \Delta_{01} u_{mn} = O(1)$$

are Tauberian conditions for the $(\overline{N}, p, q)$ summability under some additional conditions.

Keywords— Double sequences, convergence in Pringsheim's sense, the $(\overline{N}, p, q)$ summability, regularly varying sequences, slowly oscillating sequences, Tauberian conditions and theorems

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