Functions holomorphic over finite-dimensional commutative associative unital $\mathbb{C}$-algebras

Marin Genov

Institute of Mathematics and Informatics, Bulgarian Academy of Sciences

marin.genov@math.bas.bg

Let $\mathcal{A}$ be a finite-dimensional commutative associative unital $\mathbb{R}$-algebra and let $U \subseteq \mathcal{A}$ be an open subset. A function $f\colon U \to \mathcal{A}$ is called $\mathcal{A}$-differentiable at the point $Z_0 \in U$ iff

$$f'(Z_0) := \lim_{\substack{H \to 0 \\ H \in \mathcal{A}^\times}} \frac{f(Z_0 + H) - f(Z_0)}{H}$$

exists. When $\mathcal{A}$ carries a compatible complex structure, $\mathcal{A}$-*holomorphic* functions exhibit a theory very similar to the classical theory of a single complex variable despite being functions of several complex variables. In particular, $\mathcal{A}$-holomorphic functions admit direct generalizations of the four pilars of one-dimensional Complex Analysis:

- generalized Cauchy-Riemann PDEs with *complex* coeffcients;
- *single-variable* Cauchy and Morera Integral Theorems;
- a *single-variable* Homological Cauchy Integral Formula;
- analyticity over $\mathcal{A}$ in the $\mathcal{A}$-variable Z.

Time permitting, we then go on to briefly discuss

- the relationship between algebra structures $\mathcal{A}$ on $\mathbb{C}^n$ and their corresponding $\mathcal{A}$-holomorphic functions,
- a corresponding *homology* notion,
- convergence behavior and the algebra $\mathcal{A}\{Z\}$,
- canonical $\mathcal{A}$-holomorphic continuations,
- a characterization of the corresponding structure sheaf $\mathcal{O}_{\mathcal{A}}$,
- $\mathcal{A}$-meromorphic functions and their singularities,
- a corresponding Residue Theorem,
- the algebra $\mathcal{A}\{\{Z\}\}$,

not necessarily in that order, etc. Finally, we give some canonical examples of “Riemann Surfaces” over $\mathcal{A}$.