Bivariate Bernstein-gamma functions and asymptotic behaviour of exponential functionals on deterministic horizon

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For almost thirty years, researchers have been studying the properties of classical exponential functionals of Lévy processes [2]. We will introduce the uni- and bivariate Bernstein-gamma functions [1, 3], which are intricately connected to exponential functionals through their Mellin transform and further explore this connection to obtain asymptotic results on deterministic horizon.

While there are many results on infinite horizon, there are fewer asymptotic results on fixed horizon and there appears to be no unified approach to the study of these objects. Our method aims to address this issue by investigating this problem more broadly by using a wide range of techniques, including Mellin inversion, Tauberian theorems (including de Haan theory), various properties of Bernstein-gamma function and their link with potential theory. As a result, we obtain that if X is a heavy-tailed Lévy process which is not a compound Poisson process, drifts to $-\infty$ and $\mathbb{P}(X_1 \geq t) \overset{\infty}{\sim} \ell(t)/t^{\alpha}$ for a slowly varying at infinity ℓ and $\alpha > 1$, then for every $c \in (0,1)$

$$\frac{t^{\alpha}\mathbb{P}\left(\int_0^t e^{-X_s}\mathrm{d}s \in \mathrm{d}y\right)}{y^c \ell(t)} \xrightarrow[t\to\infty]{w} \nu_c(\mathrm{d}y)$$

with an explicit ν_c, which strongly extends a recent result of Xu [4].

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