

CHARACTERIZATION OF EXCHANGEABLE MEASURE-VALUED PÓLYA URN SEQUENCES

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Measure-valued Pólya urn sequences (MVPS) are a generalization of the observation processes generated by k -color Pólya urn models, where the space of colors \mathbb{X} is a complete separable metric space and the urn composition is a finite measure on \mathbb{X} , in which case reinforcement reduces to a summation of measures. Here, we provide a representation theorem for the reinforcement measures R of all exchangeable MVPSs, which leads to a characterization result for their directing random measures \tilde{P} . When \mathbb{X} is countable or R is dominated by the initial distribution ν , then any exchangeable MVPS is a Dirichlet process mixture model over a family of probability distributions with disjoint supports. Furthermore, for all exchangeable MVPSs, the predictive distributions converge on a set of probability one in total variation to \tilde{P} . A final result shows that \tilde{P} can be decomposed into an absolutely continuous and a mutually singular measure with respect to ν , whose support is universal and does not depend on the particular instance of \tilde{P} .

Keywords: Exchangeability; Reinforced processes; Urn models; Directing random measures; Pólya sequences; Dirichlet process mixture.