CHARACTERIZATION OF SOME STRONG $(t \mod q)$-ARCS

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Abstract

An arc $\mathcal{K}$ in $\mathrm{PG}(r,q)$ is called a $(t \mod q)$-arc if $\mathcal{K}(L) \equiv t \pmod q$ for every line L in $\mathrm{PG}(r,q)$. If the maximal multiplicity of a point is t then $\mathcal{K}$ is called a strong $(t \mod q)$-arc. Strong $(t \mod q)$-arcs can be obtained by the so-called lifting construction from $(t \mod q)$-arcs in geometries of smaller dimension [3, 5]. Arcs obtained by this construction are called lifted arcs. Until recently, the only known strong $(t \mod q)$-arcs for dimensions $r \geq 3$ were lifted. In [3] three strong non-lifted $(3 \mod 5)$-arcs in $\mathrm{PG}(3,5)$ are constructed of respective sizes 128, 143, 168. The first one is related to the exceptional 20-cap in $\mathrm{PG}(3,5)$ of Abatangelo, Korchmaros and Larato [1]. The other two are related, respectively, to the elliptic and to the hyperbolic quadric in $\mathrm{PG}(3,5)$.

In this note we present a generalization of the construction using quadrics to geometries of larger dimension over larger fields of odd characteristic. Let $f(X_0,\ldots,X_r)$ be a quadratic form in $r+1$ variables. The arc $\mathcal{K}$ defined by

$$\mathcal{K}(P) = \begin{cases} \frac{q+1}{2} & \text{for } f(P)=0, \\ 1 & \text{for } f(P) \text{ a square/non-square}, \\ 0 & \text{for } f(P) \text{ a non-square/square}. \end{cases}$$

is a strong $(t \mod q)$-arc in $\mathrm{PG}(r,q)$ which is not lifted [4]. Arcs obtained by this construction are called quadratic $(t \mod q)$-arcs in analogy with quadratic sets of Buekenhout [2]. It turns out that for the fields with three ot five elememnts these are the only $(t \mod q)$-arcs.

Theorem 1. *Every strong $(t \mod q)$-arc in $\mathrm{PG}(r,3)$, for $r \geq 2$, or in $\mathrm{PG}(r,5)$, for $r \geq 4$, is either lifted or a quadratic $(t \mod q)$-arc.*

It is conjectured that for any odd prime power q there exists a positive integer r_0 such that for all $r \geq r_0$ every strong $(t \mod q)$-arc in $\mathrm{PG}(r,q)$ is either lifted or quadratic.

References

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