Multiplicities for Smoothable Curves

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A core problem of equisingularity consists of finding conditions for two singular germs to be similar in several categories: we work and relate topological, differential, analytic or formal equivalences of analytic germs. The problem in general seems beyond reach, but when considered in families, we can hope to approach the problem numerically. Let $f : (X, 0) \to (Y, 0)$ be such a (flat) family, then our goal is to show that the constancy of some numerical invariants of the fibres $X_y = f^{-1}(y),\ y \in Y$ implies the equivalence of these fibres.

In my talk I will give a brief overview of a work in collaboration with A. Rangachev, relating the variation of multiplicities along the fibres, to local invariants of the specialization of curves. By computing intersection numbers with the polar variety associated to $(X, 0)$, one finds expressions relating Buchsbaum-Rim multiplicities of the Jacobian modules of the fibres to the Milnor number of the special fibre. We introduce a correction term that vanishes for complete intersections that can be found in T. Gaffney's work. Our work builds on his contributions to extend the principle of specialization of integral dependence to smoothable curves (*i.e.,* curves that are specializations of smooth curves).